Prediction of Fatigue Crack Growth at Cold-Worked Fastener Holes

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A methodology for the quantitative prediction of crack-growth behavior of cracks emanating at cold-worked fastener holes under fatigue cyclic loading has been developed. The proposed prediction technique is based on an "effective stress field" concept that accounts for the amount of the compressive residual stress existing at the edge of the cold-worked hole. Stress-intensity factor ranges (ΔK) and crack-growth rates (da/dn) are all formulated in terms of the effective stress field. An existing fatigue-crack-growth analysis computer program has been modified to account for these changes. This program was subsequently used to study the fracture-mechanics design development test data. Good correlations have been obtained.

Nomenclature

a	= crack depth
a_i	= initial crack depth
a_{cr}	= critical crack depth
da/dn	= cyclic crack-growth rate
c	= crack length
c_i	= initial crack length
c_{cr}	= critical crack length
C,m,n	= growth constants
D	= diameter of hole
F_c	= correction factor
F_{ty}	= material tensile yield strength
F_{tu}	= material tensile ultimate strength
K	= stress intensity factor
K_I	= stress intensity factor, mode I
ΔK	= stress intensity factor range, $K_{max} - K_{min}$
ΔK_{th}	= threshold stress intensity factor range
k_t	= elastic stress concentration factor
$\bar{K}$	= effective stress intensity factor
$\Delta\bar{K}$	= $\bar{K}_{max} - \bar{K}_{min}$
M	= geometric function
Q	= cold-expansion level
$\bar{R}$	= effective stress ratio ($\bar{\sigma}_{min}/\sigma_{max}$)
R_{cut}	= cutoff value of R
t	= plate thickness
σ_∞	= remotely applied tensile stress
σ_{max}	= maximum cyclic stress
σ_{min}	= minimum cyclic stress
σ_{res}	= residual stress
σ_θ	= tangential stress
$\bar{\sigma}$	= effective stress
$\bar{\sigma}_{max}$	= maximum effective stress
$\bar{\sigma}_{min}$	= minimum effective stress
Φ	= elliptical shape normalizing factor
ξ	= aspect ratio

Introduction

INVESTIGATIONS on aircraft structural failures have indicated that flaws originating at fastener holes were one of the most prevalent sources of fracture in aircraft structures.[1] Among various techniques developed to date for fatigue enhancement of fastener holes is the application of the cold-working process. Fatigue lives of cracked fastener holes have been shown to be substantially increased through the cold-working operation.[2,3] Figure 1 shows a typical comparison of crack growth behavior at a cracked conventional open hole and a cracked cold-worked hole.

Current Air Force durability and damage tolerance requirements[4,5] levied on presently developed and future air vehicles to consider fracture control procedures have necessitated the development of advanced analytical methods to evaluate the crack growth behavior of flaws and cracks in primary structures under flight loading. To date, analytic methods reported in the literature relating to the cold-worked fastener hole situations have been limited to the calculation of the stress distribution around an initially cold-worked hole in a plate[6] and the determination of the mode I stress intensity factor (K_I) of a through-crack at a cold-worked fastener hole.[7,8] Corresponding crack growth prediction methods are totally lacking. Recently, an analysis procedure was developed through an independent R&D program and a fracture-mechanics test data correlation study.[9] This prediction methodology will provide aircraft designers and analysts an engineering tool with which to conduct durability and damage tolerance analyses and prepare fracture control procedures on structures containing cold-worked holes.

Stress-Intensity Factor Solutions

The proposed prediction methodology on fatigue crack-growth behavior is based on the principles of linear elastic fracture mechanics (LEFM). In the concepts of LEFM, the primary cause of fatigue crack growth is the cyclic variation in the local stress field around the crack tip. This variation is characterized by the stress intensity factor range, ΔK. Thus, to predict the cyclic growth behavior of flaws growing from cold-worked holes, the crack tip stress intensity factor distribution in the residual stress-affected zone must be determined. A precise determination of the stress intensity factor for a flawed body would require a rigorous analytic solution. For the cold-worked hole case, the exact solution is very difficult, if not impossible, to obtain. However, an "effective" stress intensity factor concept,[10] suitable for engineering practice, was adopted. The effective stress intensity factor $\bar{K}$ is formulated in terms of an effective stress $\bar{\sigma}$. In the case of two corner cracks emanating from the hole, it can be expressed as

$$\bar{K} = M_F(a/c) M_B(a/c, a/t)\bar{\sigma}\sqrt{\pi a/Q}$$

where a is the crack depth, c is the crack length, t is the structural thickness, and M_F and M_B are the front-face and

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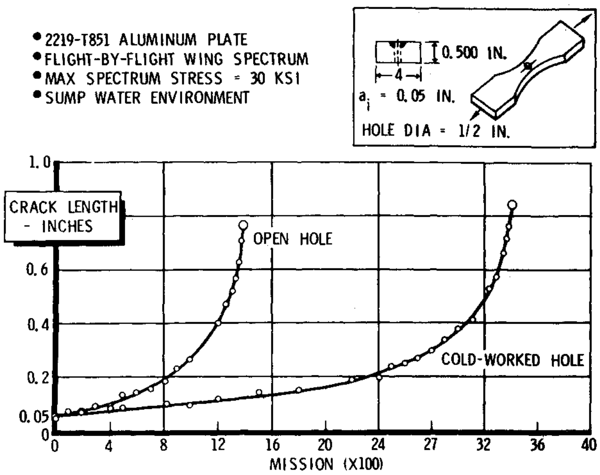

Fig. 1 Fatigue crack growth comparison, cold-worked vs conventional hole.

back-face correction factors, respectively. The front face correction factor suggested by Kobayashi and Moss[11] is employed. Table 1 shows the front-face correction factor at different $a/2c$ values. The back-face correction factors can be derived from the surface magnification factor given by Kobayashi and Shah.[12] Table 2 contains the back-face correction factor for various a/t and a/c ratios.

The quality Q is a combined factor expressed in terms of the elliptic shape normalizing factor Φ and the ratio of the applied stress to the material yield strength ($\bar{\sigma}/\sigma_{ys}$) as proposed by Irwin.[13]

$$Q=\Phi^2-0.212(\bar{\sigma}/\sigma_{ys})^2$$

Values of Φ are the complete elliptic integral of the second kind, which can be expressed as

$$\Phi=\int_0^{\pi/2}\sqrt{1-(1-4\xi^2)\sin^2\theta}\ \mathrm{d}\theta \qquad \xi=\frac{a}{2c}$$

Effective Stresses

Residual stresses are generated at the periphery of fastener holes cold-worked by such techniques as the mandrel expansion method. The effective stress is defined to account for the residual stress. For a plate containing a cold-worked hole under a remote tensile stress σ_∞, the effective stress is expressed as

$$\bar{\sigma}=(F_c\sigma_\infty+\sigma_{res})$$

where F_c is a correction factor that accounts for the influence of the hole to the local stress field and σ_{res} is the tangential residual stress, either in compression or tension.

For a cold-worked open hole contained in a wide plate, Bowie's[14] stress-intensity correction factor for through-cracks emanating at an open hole contained in a infinite plate can be used. In other cases, such as a cracked open hole contained in a narrow plate, the correction factor can be assumed to be

$$F_C=1.12k_t$$

where k_t is the corresponding stress concentration factor of the uncracked hole. The constant 1.12 is a stress intensity correction factor for a single-edge crack.[15] This is to assume that when the crack is small, compared to the hole diameter,

Table 1 Front-face correction factors

a/c	0	0.1	0.2	0.4	0.6	0.8	1.0
M_F	1.12	1.11	1.1	1.08	1.06	1.04	1.05

D (i.e., $a/D\leq 1$), the edge of the hole can be considered to be a free edge. The error was calculated to be less than 10%.[10]

Various techniques can be used to determine the residual stress in the immediate neighborhood of a cold-worked hole. Adler and Dupree[6] employed finite-element methods to obtain the distribution of residual stresses. An elastic-plastic finite-element computer routine has been developed to determine the influence of different degrees of cold working on the stress distribution in the vicinity of the fastener hole. For engineering approximation, a much simpler closed-form solution can be employed. This is the elastoplastic solution of a pressurized thick-walled cylinder.[16] The residual stresses are obtained by calculating the stresses in the plastic and elastic regions due to the mandrel cold expansion and subtracting from these values the "equivalent" elastic stress that would prevail if the material were elastic at the applied pressure. The procedure used to calculate the residual stress is presented in the Appendix. A typical tangential residual stress distribution determined from the closed-form solution is shown in Fig. 2. It shows that, after a 0.5-in.-diam hole in an aluminum plate is cold expanded 0.007 in., a compressive residual stress as high as −46 ksi exists in the immediate neighborhood of the edge of the hole. The magnitude of the compressive stress decays drastically. At a distance approximately equal to the diameter of the hole, the residual stress becomes tension. It is the compressive residual stress that delays the fatigue growth of the crack. A comparison of the closed-form solution to the finite-element solution obtained by Adler and Dupree[6] is shown in Fig. 3.

For cyclic loading, the maximum and minimum effective stresses are to be determined in terms of the residual stress as follows

$$\bar{\sigma}_{max}=(F_c\sigma_{max}+\sigma_{res})$$

$$\bar{\sigma}_{min}=(F_c\sigma_{min}+\sigma_{res})$$

where σ_{max} and σ_{min} are the maximum and minimum values of the remotely applied cyclic stresses.

Fatigue Crack-Growth Rate Model

A Paris-type fatigue crack growth rate equation[17] with the Walker stress ratio correction[18] was employed. In terms of the effective stress intensity factor range ΔK, the fatigue crack growth equation for the cracked cold-worked hole case can be expressed as

$$\frac{\mathrm{d}a}{\mathrm{d}n}=C[(1-\bar{R})^{(m-1)}\Delta\bar{K}]^n$$

where C, m, and n are experimentally determinable constants.

The effective stress ratio $\bar{R}$ in the modified rate equation is defined as

$$\bar{R}=\bar{\sigma}_{min}/\bar{\sigma}_{max}$$

where $\bar{\sigma}_{max}$ and $\bar{\sigma}_{min}$ are the maximum and minimum effective stresses.

In terms of the residual stresses, the effective stress ratio can be written as

$$\bar{R}=\frac{(F_c\sigma_{min}+\sigma_{res})}{(F_c\sigma_{max}+\sigma_{res})}$$

The modification of the growth rate equation to account for the residual stress in both the stress intensity and the stress ratio calculation plays the key role in this proposed fatigue crack growth prediction methodology. Figure 4 illustrates schematically the changes of the peak-stress and the stress-ratio values due to the existence of the compressive residual stresses. The reduction of the peak cyclic stress and stress ratio has long been known as the major causative factor in the fatigue life improvement of cold-worked fastener holes.[19]

Table 2 Back-face correction factors

a/c \ a/t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.0	1.02	1.04	1.05	1.08	1.13	1.2	1.36	1.76
0.2	1.0	1.0	1.02	1.03	1.06	1.1	1.16	1.27	1.53
0.4	1.0	1.0	1.0	1.01	1.04	1.07	1.13	1.20	1.37
0.6	1.0	1.0	1.0	1.01	1.02	1.05	1.09	1.16	1.28
0.8	1.0	1.0	1.0	1.01	1.02	1.04	1.07	1.13	1.24
1.0	1.0	1.0	1.0	1.0	1.01	1.02	1.05	1.10	1.19

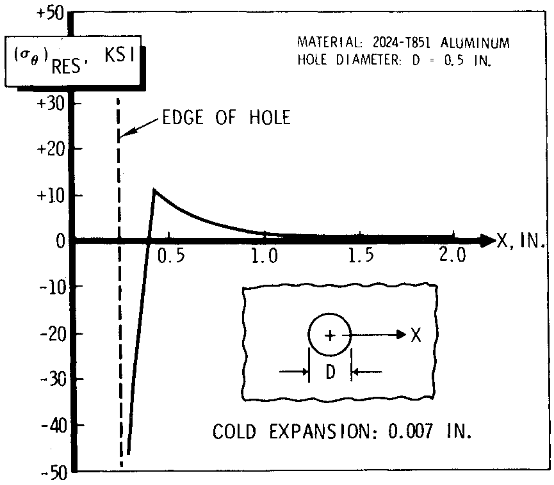

Fig. 2 Residual tangential stress profile of a cold-worked fastener hole in 2024-T851 aluminum plate.

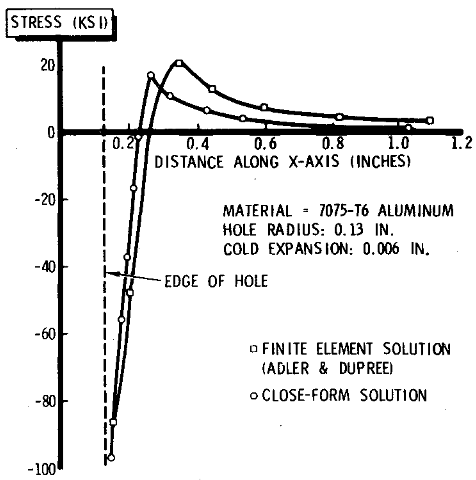

Fig. 3 Tangential residual stress distribution after cold expansion.

Test Data Correlations

Fatigue crack growth test data for cold-worked fastener holes from a fracture mechanics design development test program have been employed to check the proposed crack growth prediction methodology. A typical test specimen geometry and crack configuration are shown in Fig. 5. The specimen thickness was 0.5 in. The Boeing mandrel expansion process was used for the application of cold-working. Figure 6 shows a schematic presentation of this cold expansion process, i.e., pulling a mandrel through a split sleeve inserted in the hole. After pulling the mandrel, the sleeve is removed and the hole is reamed to the final size. Initial radial corner flaws were introduced by the electrical discharge machining (EDM) process and the specimens were then fatigue precracked under bending loads. Flight-by-flight spectrum loading was applied. Crack growth was visually monitored,

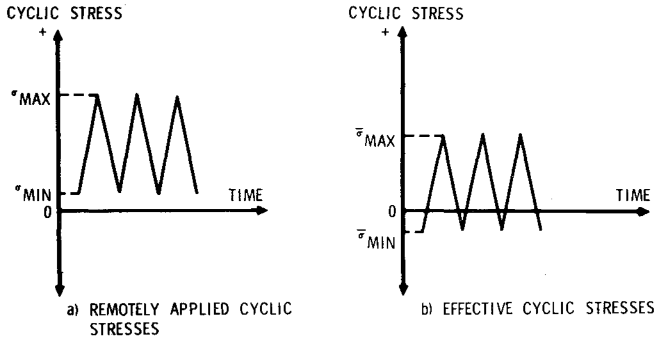

Fig. 4 Schematic of residual compressive stress effect to cyclic loading due to cold-working.

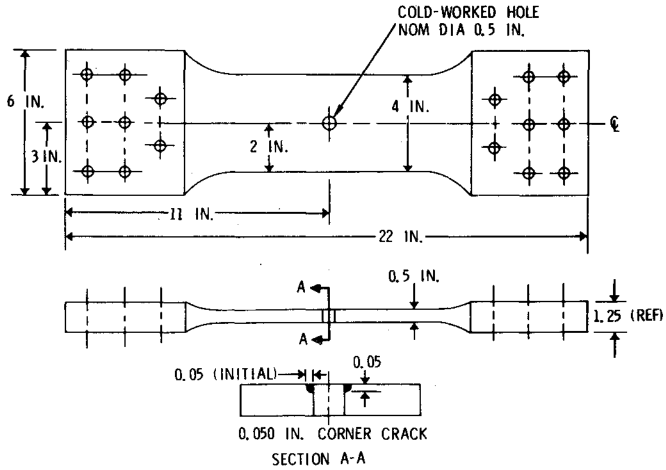

Fig. 5 Specimen geometry and crack configuration.

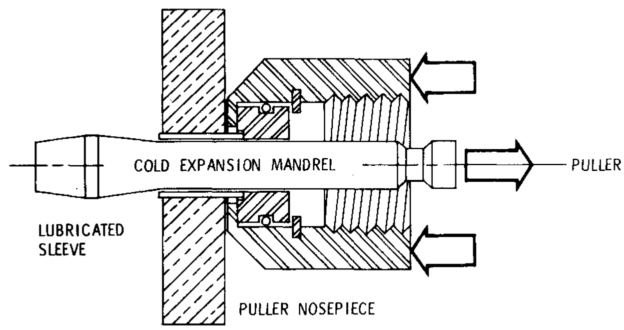

Fig. 6 Split-sleeve cold-expansion process schematic.

and measurements were recorded and plotted. Figure 7 shows a typical plot of crack growth at a preflawed cold-worked hole in a 2024-T851 aluminum plate.

An existing fatigue crack growth analysis computer routine, EFFGRO,[20] has been modified for the prediction of the growth behavior of cracks originating at cold-worked holes. Modifications were made to account for the residual stresses in both the stress intensity factor and stress ratio calculation. Fatigue crack growth data and other material properties used in the analyses were as shown in Figs. 8 and 9. Crack growth behavior predicted through the use of this computer program was compared to the test data obtained from a fracture mechanics test program. Reasonably good correlations were obtained. Figures 10 and 11 show typical correlations. In both cases, a 0.5-in.-diam hole in a 4.0-in.-wide aluminum plate was cold-expanded to a level, $Q = 0.007$ in. Two radial corner cracks with aspect ratio $a/2c = 0.5$ were introduced and precracked. The flight-by-flight spectra were applied to the specimens in a sump tank water environment. The solid lines

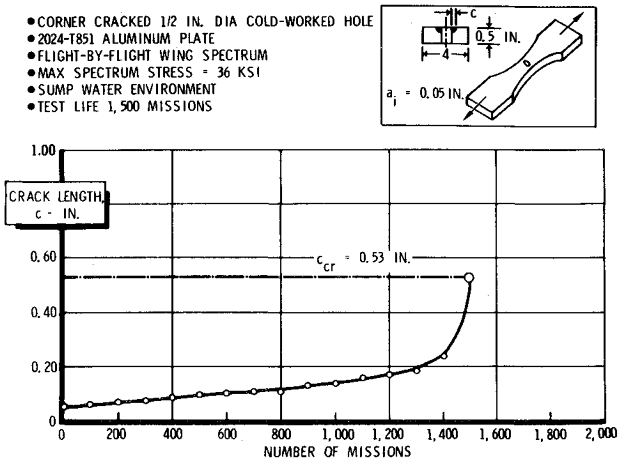

Fig. 7 Test data: Fatigue crack growth at a cold-worked hole.

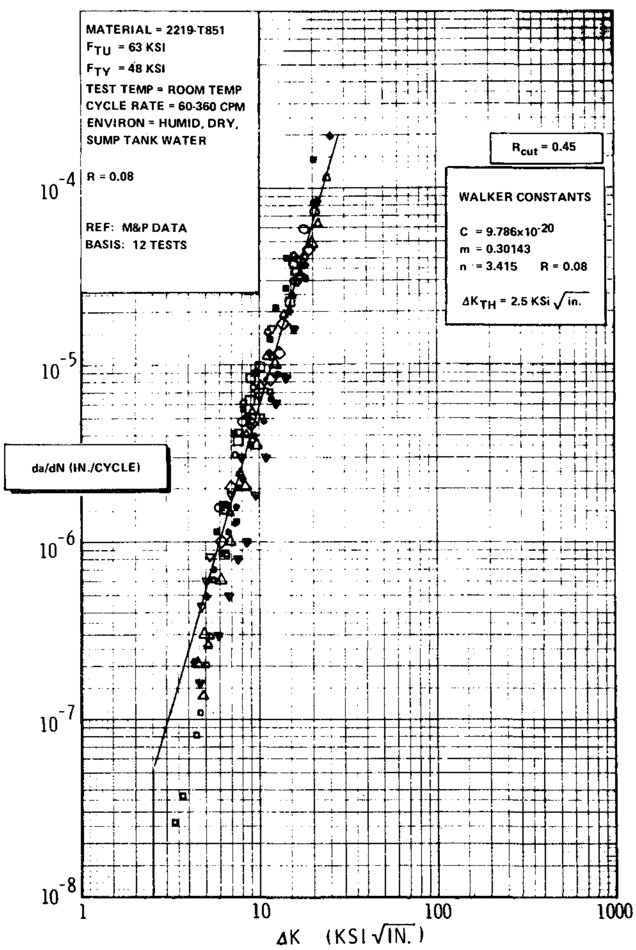

Fig. 8 Material and fracture properties of 2219-T851 aluminum.

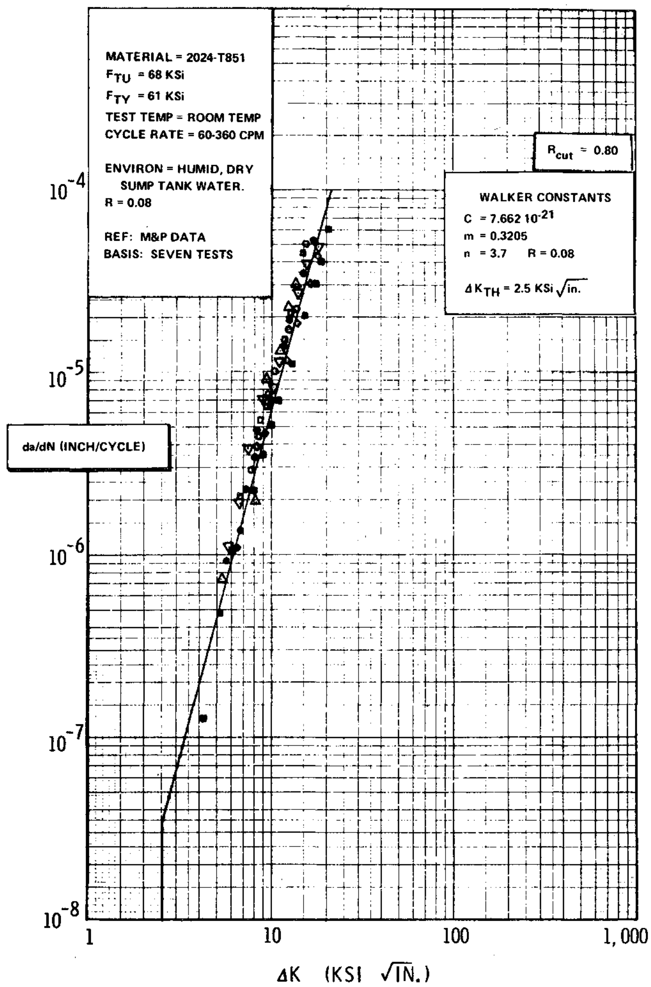

Fig. 9 Material and fracture properties of 2024-T851 aluminum.

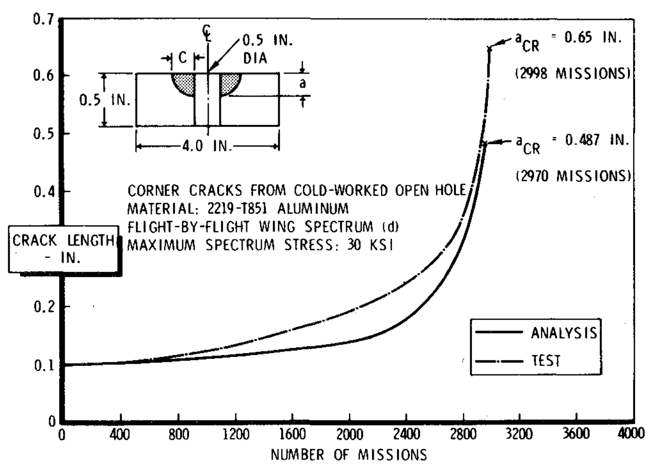

Fig. 10 Crack growth data correlation, cold-worked open hole in a 2219-T851 aluminum plate.

in the figures represent the growth behavior predicted by the proposed methodology. The dotted lines represent the test data. They show that the largest errors of life prediction in these two cases are within 15%.

Concluding Remarks

A prediction methodology for fatigue crack growth behavior of flaws emanating from cold-worked fastener holes has been presented. The proposed methodology was developed based on the linear elastic fracture mechanics concept. Although the question has often arisen whether the stress intensity factor is a proper fracture parameter in the presence of the large plastic strains,[21,22] correlations of test data to analytical results obtained by employing the proposed prediction methodology have indicated that reasonably good predictions still can be achieved through the application of linear elastic fracture mechanics. Other fracture parameters such as the J integral, might be another choice.[23] The uncertainty of the validity of applying the path independent integral to cyclic loading has prevented the direct usage of the J integral concept to predict fatigue crack growth at cold-worked fastener holes. Recent research performed by Dowling and Begley[24] on the extension of the J integral

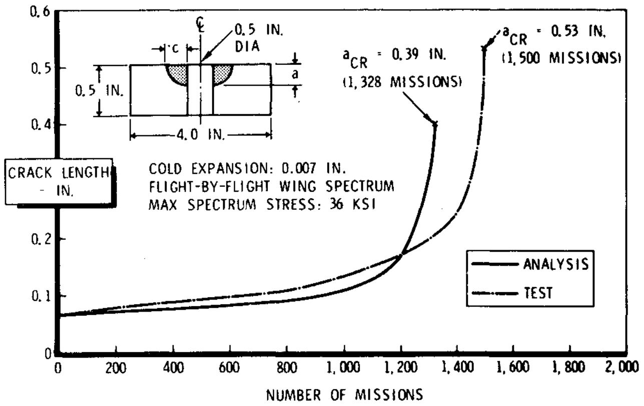

Fig. 11 Crack growth data correlation, cold-worked open hole in a 2024-T851 aluminum plate.

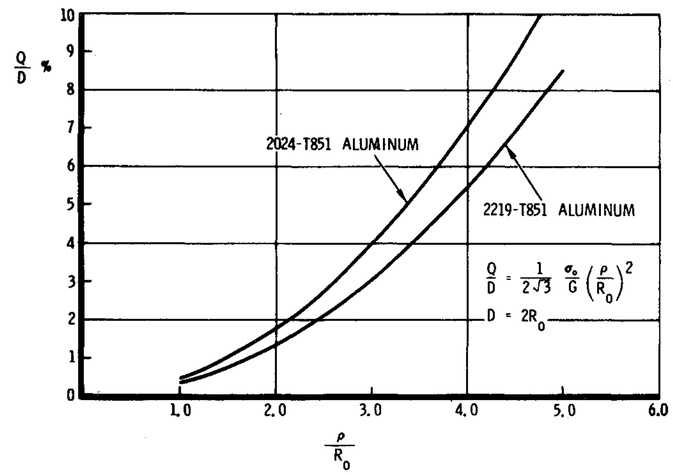

Fig. 12 Percentage of cold expansion, Q/D vs normalized radius of plastic zone ρ/R_0.

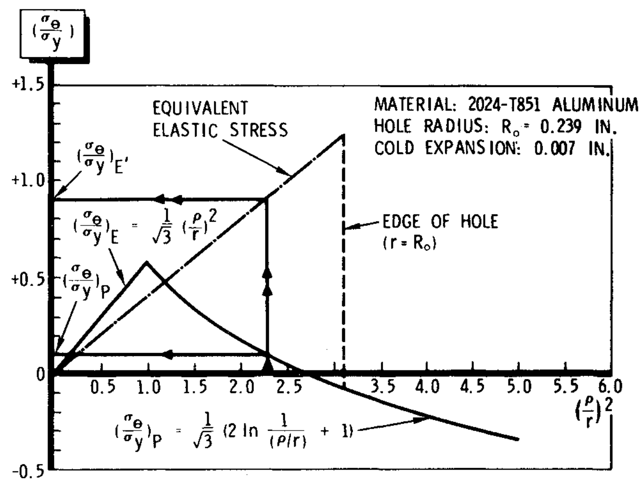

Fig. 13 Determination of tangential residual stresses of a cold-expanded fastener hole.

application to fatigue crack growth is encouraging. Future work along this direction is urged.

Appendix

The residual stress distribution adjacent to an open hole in a thick plate subjected to the mandrel expansion type of cold-working can be determined by using the elastoplastic solution of a pressurized thick-walled cylinder. Assuming the material is ideally plastic, the tangential stresses in the plastic region under the plane strain condition can be expressed as

$$\sigma_\theta = \frac{\sigma_0}{\sqrt{3}} \left(2\ln \frac{r}{\rho} + \frac{\rho^2}{R_l} + 1 \right) \tag{1}$$

Where σ_0 is the material yield strength, R_l is the exterior radius of the cylinder, ρ is the radius of the plastic zone, and r is the radial distance measured from the centerline of the cylinder. For the elastic region ($\rho \leq r \leq R_l$), the expression for the tangential stress is

$$\sigma_\theta = \frac{\sigma_0}{\sqrt{3}} \left(\frac{\rho^2}{R_l^2} + \frac{\rho^2}{r^2} \right) \tag{2}$$

In the case of an open hole in a wide plate, the ratio of the radius of the plastic zone ρ to the external radius of the cylinder R_l is a small value. Thus, in Eqs. (1) and (2), the $(\rho/R_l)^2$ term can be neglected. The plastic and elastic tangential stresses for a wide plate containing a hole subjected to cold expansion can then be expressed as

$$\sigma_\theta = \frac{\sigma_0}{\sqrt{3}} \left(2\ln \frac{r}{\rho} + 1 \right) \quad \text{(plastic)} \tag{3}$$

and

$$\sigma_\theta = \frac{\sigma_0}{\sqrt{3}} \left(\frac{\rho}{r} \right)^2 \quad \text{(elastic)} \tag{4}$$

The radial displacement U of the points of the plastic front has the solution

$$U = \frac{\sigma_0 \rho^2}{2\sqrt{3} G R_0} \tag{5}$$

where G is the material shear modulus and R_0 is the inner radius of the cylinder.

The amount of cold-working, Q, can be thus defined as

$$Q = 2U = \frac{1}{\sqrt{3}} \frac{\sigma_0}{G} \left(\frac{\rho^2}{R_0} \right) \tag{6}$$

Based on the elastoplastic solutions, a graphic procedure for obtaining the residual stress distribution adjacent to a cold-expanded hole has been developed.

1) To plot the percentage of the cold-expansion Q/D against the normalized radius of the plastic front ρ/R_0. This can be done by rewriting Eq. (6) as

$$\frac{Q}{D} = \frac{1}{2\sqrt{3}} \frac{\sigma_0}{G} \left(\frac{\rho}{R_0} \right)^2 \tag{7}$$

A chart of (Q/D) vs (ρ/R_0) can be plotted. Figure 12 shows the plots of 2024-T851 and 2219-T851 aluminum alloys. If the cold expansion level of a given fastener hole size is known, Q/D can be calculated. Then, from the chart, the related value of (ρ/R_0) can be determined and, hence, the radius of the plastic zone ρ.

2) To plot the normalized tangential stress σ_θ/σ_0 as a function of $(\rho/r)^2$. From Eqs. (3) and (4), the normalized plastic and elastic tangential stresses are

$$\left(\frac{\sigma_\theta}{\sigma_0} \right)_p = \frac{1}{\sqrt{3}} \left[2\ln \frac{1}{(\rho/r)} + 1 \right] \tag{8}$$

and

$$\left(\frac{\sigma_\theta}{\sigma_0} \right)_E = \frac{1}{\sqrt{3}} \left(\frac{\rho}{r} \right)^2 \tag{9}$$

Solid lines in Fig. 13 represent the elastoplastic solutions. Curve A represents the plastic stress field, whereas Curve B represents the elastic stress field. The broken line in the figure represents the equivalent elastic stress solution. At any radial distance r from the centerline of the hole, the value of $(\rho/r)^2$ can be calculated from the previously determined radius of plastic zone ρ. The corresponding residual stresses are obtained by subtracting the equivalent elastic stresses from the plastic stresses.

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